THE APRIL MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

A REGULAR meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Saturday, April 25, 1903, extending through the usual morning and afternoon sessions. About fifty persons were in attendance, including the following forty members of the Society.

Dr. Grace Andrews, Dr. C. L. Bouton, Professor E. W. Brown, Mr. J. S. Brown, Professor F. N. Cole, Miss E. B. Cowley, Professor E. S. Crawley, Dr. W. S. Dennett, Dr. L. P. Eisenhart, Dr. William Findlay, Professor H. B. Fine, Professor T. S. Fiske, Miss Ida Griffiths, Professor G. H. Hallett, Miss Carrie Hammerslough, Professor James Harkness, Dr. H. E. Hawkes, Dr. J. I. Hutchinson, Dr. S. A. Joffe, Dr. Edward Kasner, Professor C. J. Keyser, Mr. L. L. Locke, Professor James Maclay, Dr. C. R. Mann, Professor E. H. Moore, Professor W. F. Osgood, Dr. I. E. Rabinovitch, Miss I. M. Schottenfels, Dr. Arthur Schultze, Dr. W. M. Strong, Mr. John Tatlock, Jr., Professor H. W. Tyler, Professor E. B. Van Vleck, Professor J. M. Van Vleck, Mr. H. E. Webb, Professor A. G. Webster, Dr. J. K. Whittemore, Miss E. C. Williams, Dr. Ruth G. Wood, Professor R. S. Woodward.

The President of the Society, Professor Thomas S. Fiske, occupied the chair. The Council announced the election of the following persons to membership in the Society: Professor W. N. Ferrin, Pacific University, Forest Grove, Ore.; Mr. E. H. Koch, Jr., Mackenzie School, Dobbs Ferry, N. Y.; Professor N. C. Riggs, Armour Institute of Technology, Chicago, Ill.; Mr. K. D. Swartzel, Harvard University, Cambridge, Mass. Thirteen applications for admission to the Society were received.

The following papers were read at this meeting:

(1) DR. H. E. HAWKES: "On non-quaternion number systems in seven units."

(2) Professor B. O. PEIRCE: "On families of curves which are the lines of certain plane vectors, either solenoidal or lamellar."

(3) Professor E. W. BROWN: "On the variation of the arbitrary and given constants in dynamical equations."

(4) Dr. L. P. EISENHART: "Congruences of tangents to a surface, and derived congruences."

(5) Dr. H. F. STECKER: "Least distance in the non-euclidean plane."

(6) Professor L. E. DICKSON: "Fields whose elements are linear differential equations."

(7) Dr. SAUL EPSTEEN: "On linear differential congruences."

(8) Professor R. S. WOODWARD: "The deviation from the vertical of falling bodies" (preliminary communication).

(9) Dr. EDWARD KASNER: "The automorphic groups of the manifolds defined by a general and a symmetric determinant."

(10) Mr. C. H. SISAM: "On some directrix curves on quintic scrolls."

(11) Mr. L. I. NEIKIRK: "Groups of order p^m which contain cyclic subgroups of order p^{m-3}."

(12) Miss I. M. SCHOTTENFELS: "On the simple groups of order $8!/2$."

(13) Dr. E. B. WILSON: "The so-called foundations of geometry."

Mr. Sisam's paper was communicated to the Society through Dr. Snyder; Dr. Epsteen's through Professor Dickson. Mr. Neikirk was introduced by Professor Crawley. In the absence of the authors, Mr. Sisam's paper was read by Dr. Hutchinson, and the papers of Professor Peirce, Dr. Stecker, Professor Dickson, Dr. Epsteen and Dr. Wilson were read by title.

Professor Dickson's paper will be published in the BULLETIN. Abstracts of the other papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. At the December meeting of the Society Dr. Hawkes gave a method for the enumeration of all non-quaternion number systems with skew units. The present paper applies this method to the case $n = 7$ and deduces expeditious means of carrying out further enumerations should it be desirable.

2. If $u \equiv f(x, y) = c$ represents any family of curves in the xy plane, any vector V the components of which are of the form

$$\left(F(x, y), -F(x, y) \cdot \frac{\partial u}{\partial x} \Big/ \frac{\partial u}{\partial y}, 0\right)$$

where F is arbitrary, evidently has the u curves for its lines. Of all the vectors which have these given lines, an infinite number are in general solenoidal and an infinite number are lamellar, but no one can be both solenoidal and lamellar, unless u happens to satisfy Lamé's well known condition for an isothermal parameter $[\nabla^2(u)/h_u^2$ a function of u only], in which case there are a set of such vectors connected by a linear relation.

In certain general problems in mathematical physics, classes of lamellar or solenoidal vectors appear which are known to satisfy in each case some general condition common to all of the class. Sometimes, for instance, the tensor, or the divergence, or the tensor of the curl of a vector must be constant in a given region; or the tensor, or the divergence, or the curl must be expressible in terms of the parameter of the lines of the vector, or in terms of the parameter of the orthogonal lines. Professor Peirce's paper discusses briefly the possible lines of plane vectors which satisfy a few of the more common conditions. For example, it is shown that, if $u = c$, $v = k$ are the equations of curves in the xy plane which cut each other orthogonally, the condition that the u curves shall be possible lines of a set of solenoidal vectors, the tensors of which involve u only, is the condition that the v curves shall be possible lines of a set of lamellar vectors, the tensors of which involve u only. If the u curves are possible lines of a solenoidal vector, the tensor of which is expressible in terms of v, the v curves are the lines of a lamellar vector, the tensor of which is a function of v only. If a solenoidal vector which has the u curves for lines has a tensor expressible in terms of u, the u curves must form a parallel system, etc., etc.

3. Professor Brown's paper contains certain results with reference to the arbitrary constants which arise in the integration of a set of dynamical equations and also with reference to the constants which occur in the force function. The equations are supposed to have been solved by trigonometric series with arguments some of which are arbitrary and some of which are present in the force function. In the latter part of the paper it is considered under what circumstances we may substitute a variable value for a constant as well in the solution as in the force function. It then appears that Newcomb's well known theorem on the secular accelerations is a particular case of sub-

stitution of a variable for a constant in the solution of a set of such differential equations.

4. Given a family of curves upon a surface S; the tangents to these curves form a rectilinear congruence for which S is one of the focal sheets. The other focal sheet is a determinate surface S_1, and upon it there is a family of curves to which the lines of the congruence are tangent. If tangents are drawn to the curves on S_1, which are conjugate to the latter, a second congruence is formed with S_1 for one focal sheet and another surface S_2 for the other sheet. This process can be continued indefinitely unless one of the surfaces reduces to a curve. In a similar manner we can get another sequence of congruences by starting with the tangents to the conjugates of the given curves on S. These are the *derived congruences* of Darboux. After finding the expressions for the different functions of these congruences in terms of functions of S, Dr. Eisenhart applies them to a study of such sequences where all the congruences of the series are of a particular form. Thus it is shown that the system of curves upon three consecutive surfaces cannot be formed of lines of curvature, so that there cannot be a sequence of congruences of Guichard, as Bianchi calls those whose developables touch the focal sheets in lines of curvature of the latter. From this it follows almost immediately that there are no series of congruences of which each member is a normal congruence.

If a congruence of Ribaucour is defined as one whose developables cut the mean surface in a conjugate system, the condition that the tangents to the curves $v = \text{const.}$ of a surface form such a congruence takes a very simple form. From this it is found that, when the tangents to the curves $u = \text{const.}$ conjugate to the former also form a congruence of this kind, all the members of the sequence are congruences of Ribaucour. A large number of surfaces are found to have a conjugate system which has this property. It is shown that the only isothermic surfaces whose lines of curvature form such a system have either of the following linear elements when referred to this system:

$$ds^2 = UV(du^2 + dv^2), \quad ds^2 = e^{UV}(du^2 + dv^2),$$

where U is a function of u alone and V is a function of v alone.

When one proceeds to the determination of the surfaces with

these linear elements, he finds that U or V must be constant, so that the only surfaces are surfaces of revolution. From this one is led to the result that the tangents to the meridians of a surface of revolution form a normal congruence of Ribaucour.

The equations of condition that the tangents to a given family of curves on S form a cyclic congruence can be brought to a very simple form, so that quite a number of theorems follow almost immediately. Among others one notes that when the tangents to the lines of curvature in one system upon a surface form a cyclic congruence, it is at the same time a congruence of Ribaucour and conversely. The discussion closes with an inquiry as to the existence of a sequence of cyclic congruences for which the circles of each congruence are equal, and it is found that there are none of this kind.

5. The problem treated in Dr. Stecker's paper is an application of the calculus of variations to determine, directly from the distance integral, the least distance between two fixed points in the non-euclidean plane.

7. In the *Comptes Rendus*, 1897, page 489, Guldberg shows that it is possible to find for linear differential forms a theory which is analogous to the Galois field theory. In the present paper Dr. Epsteen develops this theory up to the point of demonstrating the theorem: Every existent differential field of order s may be represented as a Guldberg field of order $s = p^n$; the $GuF(p^n)$ is defined uniquely by its order; in particular, it is independent of the special irreducible differential congruence used in its construction. This is the exact analogue of a well-known theorem of E. H. Moore (Chicago Congress Papers, page 220; Dickson, Linear Groups, page 14).

8. In his preliminary communication Professor R. S. Woodward reviewed the history of the problem of the deviation from the vertical of falling bodies from the time of its earliest treatment by Gauss * and Laplace,† in 1803, down to the present time. He also presented a solution of the problem differing from any of the solutions hitherto published, but giving the same results, to terms of the order of the square of the angular velocity of the earth, as those derived by Gauss, and

* *Werke*, vol. 5. pp. 495–503.
† *Mécanique Céleste*, vol. 4, chapter 5.

as those required by, though not fully worked out in, Laplace's investigation.

9. The first part of Dr. Kasner's paper is devoted to certain general principles of invariant theory, while the second is an application to the problem announced in the title. If the coefficients of a form are interpreted as homogeneous point coördinates in a space of sufficiently high dimension, then any invariant equated to zero represents a variety which is transformed into itself by a certain group of collineations. This group includes the so-called induced group of the coefficients, and in many cases coincides with it. It follows that any covariant (in the domain of the coefficients) of an invariant is also an invariant, a principle which includes for example the Aronhold process as a very special case.

The conclusion concerning the general determinant

$$\Delta \equiv |a_{ik}| = 0$$

is that its collineation group is composed of two continuous systems containing $2(n^2-1)$ parameters. The proof involves, in addition to the general principles, a geometric discussion of the manifold $\Delta = 0$ in space of n^2-1 dimension, in particular its two-fold projective generation. The case $n=2$ gives of course the well known six parameter group connected with a quadric surface.

10. Mr. Sisam's paper proved the two following theorems: 1° Every unicursal quintic scroll has three coplanar generators; the residual may be either a conic, a double directrix or a simple directrix and a fourth generator. 2° If the asymptotic lines of a (4, 1) scroll are reducible, two real and two imaginary generators issue from each real point of the four-fold line.

11. The groups of operations of order p^m which contain a cyclic subgroup of order p^{m-2} have been determined by Miller (see *Transactions*, volume 3, number 4). In the present paper Mr. Neikirk has determined the groups of order p^m which contain a cyclic subgroup of order p^{m-3}. The investigations are number-theoretic in character and are based on a division of the groups considered into classes. This division is shown to depend on the partitions of m which give the orders of the independent operations that enter into the con-

struction of the required group. Denoting the cyclic subgroup of order p^{m-3} by $\{P\}$ and the partition by $[a, b, \cdots]$, the numbers of resulting groups are given by: $[m-3, 3]$, 12 groups, 4 in which $\{P\}$ is self-conjugate, including the abelian group of this type, and 8 in which $\{P\}$ is not self-conjugate; $[m-3, 2, 1]$, $20+p$ groups in $8+p$ of which $\{P\}$ is self-conjugate; $[m-3, 1, 1, 1]$, 5 groups, in all of which $\{P\}$ is self-conjugate. The total number of these groups is therefore $37+p$.

The formal generational equations of all these types in terms of independent operators are given in the course of the investigation.

12. Miss Schottenfels's paper contains the proof that: 1° All types of simple groups of order 8!/2 or 20160, contain at least 960 conjugate subgroups of order 7. 2° No simple group of this order contains an operator where period is a multiple of 7 greater than 1.7, and all such simple groups are identical, with respect to the number of such operators or elements.

13. The first part of Dr. Wilson's paper contains criticisms on Professor Hilbert's "Grundlagen der Geometrie" which appeared in the *Mathematische Annalen*, October, 1902. The second part is occupied with the construction of some Hilbertian geometries and in particular a two-dimensional geometry in three dimensions. The paper is to appear in the *Archiv der Mathemathik und Physik*.

F. N. COLE.

COLUMBIA UNIVERSITY.

THE APRIL MEETING OF THE CHICAGO SECTION.

A regular meeting of the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY was held on Saturday, April 11, 1903, at the Armour Institute of Technology, in Chicago, opening at 9 A. M. The following members were present:

Professor W. W. Beman, Professor D. F. Campbell, Dr. J. V. Collins, Professor E. W. Davis, Professor B. F. Finkel, Mr. W. B. Ford, Professor Thomas F. Holgate, Professor A. M. Kenyon, Professor Kurt Laves, Dr. A. C. Lunn, Professor H. Maschke, Professor Malcolm McNeill, Professor E. H. Moore, Professor F. R. Moulton, Professor E. B. Skinner, Professor C. A. Waldo, Professor H. S. White, Professor B. F. Yanney, Professor J. W. A. Young, Professor A. Ziwet.

Professor Waldo was elected chairman. The following papers were read:

(1) Mr. OSWALD VEBLEN: "A set of independent axioms of geometry."

(2) Dr. F. R. MOULTON: "The conditions for the convergence of the expressions for the ratios of the triangles when developed as power series in the time intervals."

(3) Professor A. S. CHESSIN: "On the strains in a rapidly rotating disc."

(4) Dr. SAUL EPSTEEN: "On Loewy's fundamental theorem."

(5) Professor PETER FIELD: "On the form of a plane quintic curve with five cusps."

(6) Professor H. S. WHITE: "Polar triangles of a conic and triply tangent conics of a cubic."

(7) Professor H. MASCHKE: "Invariants and covariants of quadratic differential quantics in n variables."

(8) Mr. E. L. HANCOCK: "Asymptotic lines on certain surfaces of revolution."

(9) Professor ARCHIBALD HENDERSON: "On the construction of a double-six."

(10) Professor ARCHIBALD HENDERSON: "On the graphic representation of the straight lines upon the twenty-one different types of the cubic surface."

(11) Mr. O. J. FERGUSON: "Problems in projective geometry."

(12) Professor E. W. DAVIS: "Some groups in the logic of relatives."

(13) Mr. N. J. LENNES: "Theorems on the polygon and the polyhedron."

Mr. Veblen and Mr. Lennes were introduced to the Section by Professor Moore, and Professor Henderson by Professor Maschke; Mr. Ferguson's paper was presented through Professor Davis. In the absence of Professor Chessin and Mr. Hancock their papers were read by title.

The report of the committee on the requirements for the Master's degree, with mathematics as the major subject, was taken up for consideration and after discussion was again laid on the table till the next meeting.

The North Central Association of Science Teachers being in session at Armour Institute, a joint meeting was arranged with the mathematics section of this association, at which Professor W. W. Beman presided and the following papers were read:

"Introduction of physical and other problems into elementary mathematical courses, and the necessary consequences," by Professor C. E. COMSTOCK, of Bradley Polytechnic Institute.

"What is the laboratory method?" by Professor J. W. A. YOUNG, of the University of Chicago.

The discussion of these papers was opened by Miss Long, of the Lincoln, Nebraska, High School.

Abstracts of the papers presented at the Sectional meeting are as follows:

1. The aim of Mr. Veblen's paper, which is intended as a dissertation for the doctor's degree at the University of Chicago, is to present a system of axioms that are at once independent of one another and sufficient for the deduction of the euclidean geometry. The axioms are stated in terms of two fundamental symbols, point and order, and thus follow the trend of development inaugurated by Pasch and continued by Peano, rather than that of Hilbert. All other geometrical concepts, such as line, plane, space, motion, are defined in terms of point and order. In particular, the congruence relations are made the subject of definitions rather than of axioms. This is accomplished by the aid of projective geometry according to the method first given analytically by Cayley and Klein.

2. In rectangular heliocentric coördinates the functions considered in Professor Moulton's paper are

$$\frac{x_k y_l - y_k x_l}{y_k y_i - y_k x_i}, \tag{1}$$

where the x and y are defined by the differential equations

$$\frac{d^2x}{dt^2} = -\frac{k^2 x}{r^3}, \qquad \frac{d^2y}{dt^2} = -\frac{k^2 y}{r^3}. \tag{2}$$

All the singularities of x and y considered as functions of t are found and also the conditions under which the denominator of (1) can vanish. Having located all the singularities of the function, the true radius of convergence around any initial point is easily found. The problem is solved in all detail for all classes of conics, and tables are constructed giving numerical results for the various cases which can arise. The relation of the limits in this part of the theory of orbits, to those of the series used in the *Astronomical Journal*, No. 510, is given, and an exhibition of the corresponding question in Laplace's method. The paper will be published in the *Astronomical Journal*.

3. Professor Chessin finds expressions for the displacements of a particle in a thin rotating disc in terms of the coördinates of the particle and certain constants determined from the surface conditions.

5. Del Pezzo has shown how it is possible to obtain the equation of a five cusped quintic curve by inverting a quartic which has two cusps and which is inscribed in and circumscribed about the fundamental triangle. The equation of the quintic is written in the form $C_4^2 - \phi^3\phi_1 = 0$, where C_4 is a quartic and the ϕ's are conics; the constants being so chosen that x, y, z are factors of the equation. Professor Field divides the plane into regions by drawing the curves C_4, ϕ, ϕ_1, and by their aid obtains the form of the curves represented by the above equation.

6. A plane cubic may be so situated with reference to a conic as to admit an inscribed triangle, self-conjugate with respect to the conic, and having one degree of freedom. It is shown by

Professor White that the vertices of such a triangle will be contacts of a tritangential conic. There are 3 systems of such conics, corresponding to the 3 pro-Hessians of the cubic. The tritangential conics of a simply infinite system like that mentioned above are poloconics of a simple infinity of straight lines, with respect to some one of the 3 pro-Hessians. It is found that these lines form a pencil, all passing through one common point. There is thus shown the existence, under special conditions, of a covariant point in the system of a conic and a cubic.

7. Professor Maschke's paper will be published in the *Transactions*.

8. Mr. Hancock considers two surfaces of revolutions S and S_1, generated by the curves C and C_1. C_1 is formed by taking on the tangents to C a length equal to l^{-1} times the length of the tangent. The generating curves then are

$$z = f(u), \quad z_1 = (l-1)\, u_1 f'(lu_1) + f(lu_1),$$

and the equation of transformation from S to S_1, $u = lu_1$, $v = v_1$.

The condition that the asymptotic lines on S go over into the asymptotic lines on S_1 by the transformation is $D_1/D_1'' = e^3\, D/D''$. This is seen in particular when the generating curves are $z = u^{\frac{n}{n-2}} + C_2$ and $z_1 = k_1 u_1^{\frac{n}{n-2}} + C_2$. The paper will be offered to the *American Journal of Mathematics* for publication.

9. A double-six, in the Schläfli-Cayley notation, is represented by the scheme

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1' & 2' & 3' & 4' & 5' & 6' \end{pmatrix}$$

and is defined to be two systems of six lines each, such that no two lines of the same system intersect, but that each line of the one system intersects all but the corresponding line of the other system. In Professor Henderson's paper there is given a simple method of constructing a double-six, viz., if we select the equations of the five lines 1′, 3′, 4′, 5′, 6′, such that they do not intersect, but such that they are all met by the same straight line 2, whose equations are also known, then the equations of the remaining six lines 1, 3, 4, 5, 6 and 2′, are uniquely deter-

mined. Using quadriplanar coördinates and properly choosing the constants, a thread model of the configuration was very easily constructed.

10. Cayley in his Memoir on cubic surfaces (Collected mathematical papers, volume VI (1868), pages 359–455), has given a treatment of the twenty-three types of the cubic surface, the division depending upon the nature of the singularities, and not upon the reality of the lines. The equations there given are canonical forms in quadriplanar coördinates. Two of the surfaces however are scrolls, in which there is no question of the twenty-seven lines.

In Professor Henderson's second paper there is developed a simple method of graphically representing the lines on the remaining twenty-one types of the cubic surface, given by the above-mentioned canonical forms. By this method the mutual relations of the lines, as well as their relation to the fundamental tetrahedron, are shown and the results obtained are equally adapted to the construction of thread models or perspective drawings of the various configurations.

The paper was illustrated by perspective drawings in colors of the lines upon the twenty-one types of the cubic surface, and incidentally there was shown a graphic representation of the projection of the lines upon a cubic surface with a conical point into the Pascal configuration.

11. Mr. Ferguson's paper contained demonstrations of the following theorems: 1° If a range of points μ is projective to a pencil of rays S, and if through the points of μ there are drawn rays perpendicular to the corresponding rays of S, these generate a ruled surface of the fourth order. 2° In two projective sheaves of rays the common perpendiculars of corresponding rays generate a ruled surface of the sixth order.

12. Professor Davis established a one to one correspondence between the different ways in which $3, 4, \cdots, n$ relatives can be connected by logical multiplication and addition, and the substitutions on $3, 4, \cdots, n$ letters.

13. Under formal definitions of polygon and polyhedron, Mr. Lennes proved certain theorems on the partition of the plane and space.

THOMAS F. HOLGATE,
Secretary of the Section.

THE APRIL MEETING OF THE SAN FRANCISCO SECTION.

THE third regular meeting of the San Francisco Section of the AMERICAN MATHEMATICAL SOCIETY was held on Saturday, April 25, 1903, at Stanford University. The following fifteen members were present:

Professor R. E. Allardice, Dr. E. M. Blake, Professor H. F. Blichfeldt, Professor G. C. Edwards, Professor R. L. Green, Professor L. M. Hoskins, Dr. D. N. Lehmer, Dr. J. H. McDonald, Professor G. A. Miller, Dr. H. C. Moreno, Dr. T. M. Putnam, Professor Irving Stringham, Mr. L. C. Walker, Mr. A. W. Whitney, Professor E. J. Wilczynski.

A morning and an afternoon session were held, Professor Stringham acting as chairman at both sessions. Professor M. W. Haskell was elected a member of the programme committee in place of Professor E. J. Wilczynski, who will be abroad next year.

The following papers were read at this meeting:

(1) Professor E. J. WILCZYNSKI: "Invariants of systems of linear partial differential equations, and the theory of congruences."

(2) Dr. D. N. LEHMER: "Preliminary report on a table of smallest divisors."

(3) Professor H. F. BLICHFELDT: "Note on linear substitution groups of finite order."

(4) Professor R. E. ALLARDICE: "On some curves connected with a system of similar conics through three points."

(5) Dr. SAUL EPSTEEN: "Necessary and sufficient condition for the existence of invariant subgroups."

(6) Professor G. A. MILLER: "On reciprocal groups."

(7) Dr. H. C. MORENO and Professor G. A. MILLER: "On the non-abelian groups in which every subgroup is abelian."

(8) Mr. W. A. MANNING: "On the class of primitive substitution groups."

(9) Miss Ida M. SCHOTTENFELS: "Generational definition of an abstract group simply isomorphic with the simple substitution group G^{21}_{20160}."

(10) Dr. T. M. PUTNAM: "Certain subgroups of the quater-

nary linear fractional group of determinant unity, in the general Galois field."

The papers by Dr. Epsteen and Miss Schottenfels were read by Professor Wilczynski and the secretary respectively. All the other papers were read by their authors. Abstracts are given below.

1. Professor Wilczynski considers in his paper a system of two linear homogeneous partial differential equations with two independent variables x_1 and x_2

$$(1)\qquad \begin{aligned} \Omega &= ay_1 + by_2 + cz_1 + dz_2 + ey + fz = 0, \\ \Omega' &= a'y_1 + b'y_2 + c'z_1 + d'z_2 + e'y + f'z = 0, \end{aligned}$$

where $y_k = \partial y/\partial x_k$, $z_k = \partial z/\partial x_k$, $(k = 1, 2)$.

If one makes the transformations

$$(2)\quad y = a\eta + \beta\zeta, \qquad z = \gamma\eta + \delta\zeta, \qquad (a\delta - \beta\gamma \neq 0);$$

$$(3)\quad \xi_1 = f_1(x_1, x_2) \qquad \xi_2 = f_2(x_1, x_2), \qquad \left(\frac{\partial(\xi_1, \xi_2)}{\partial(x_1, x_2)} \neq 0\right),$$

where a, β, γ, δ, f_1 and f_2 are arbitrary functions of x_1 and x_2, (1) is transformed into another system of the same form. Moreover one may replace the system by

$$(4)\quad \phi\Omega + \psi\Omega' = 0, \quad \chi\Omega + \omega\Omega' = 0, \quad \phi\omega - \psi\chi \neq 0,$$

where ϕ, ψ, χ, ω are arbitrary functions of x_1 and x_2, and obtain again a system of the same form. These are the most general transformations which convert a general system (1) into another of the same form.

The author considers the functions of the coefficients of (1) which remain invariant under these transformations, and actually computes them for a reduced form.

If one considers four sets of simultaneous solutions

$$y^{(k)} = g^{(k)}(x_1, x_2) \quad z^{(k)} = h^{(k)}(x_1, x_2), \quad (k = 1, 2, 3, 4)$$

one may interpret them as the homogeneous coördinates of two points in space, the locus of each being a surface. The transformations (2), (3) and (4) merely displace these points on the line joining them and otherwise change merely the parametric

representation of the two surfaces. They leave invariant, therefore, the congruence of lines obtained by joining corresponding points of the two surfaces. The theory of a system of form (1) is therefore geometrically a theory of congruences. This leads to geometric interpretations for the vanishing of certain invariants, the determination of the developable surfaces of the congruence, its focal surfaces, etc. The properties thus obtained are all projective. But not all projective properties will appear, owing to the fact that congruences which are not mere projective transformations of each other, may belong to the same system (1). This question is investigated in the paper. All systems of form (1) may be reduced to a few simple types. The general type corresponds to a congruence with distinct non-degenerate focal surfaces. For instance, the system of partial differential equations of the theory of functions belongs to a congruence whose focal surfaces have degenerated into two distinct curves, and may be taken as the type of such systems.

The theory of the Laplace transformations of an equation of the form

$$\frac{\partial^2\theta}{\partial x\partial y} + a\frac{\partial\theta}{\partial x} + b\frac{\partial\theta}{\partial y} + c\theta = 0,$$

which is so important in the theory of surfaces, is also generalized to a system of the kind considered and finds its geometrical interpretation the same as in that older theory. It is of course connected with the theory of the covariants of such a system.

The author points out moreover that this theory can be generalized for other systems, and that geometrically other configurations, complexes, etc., may be studied in this way.

2. Dr. Lehmer's factor tables are intended to give the smallest divisors of all numbers less than ten millions. Multiples of 2, 3, 5 and 7 are not tabulated. All other numbers are of the form $210x + R$ where R is prime to 210. There are 48 values of R which give the headings for as many columns. The values of x are listed at the side. Each page of the tables contains 100 rows and 48 columns and serves to factor 21,000 numbers. The plan of the table is somewhat similar to that used by Lebesgue (Tables diverses pour la décomposition des nombres en leurs facteurs premiers. Par V. A. Lebesgue, Paris, 1864), but differs from it in several important particulars.

3. In determining the linear homogeneous substitution groups of finite order in two variables, Gordan makes use of the equation $1 + \cos\phi_1 + \cos\phi_2 + \cos\phi_3 = 0$; ϕ_1, ϕ_2, ϕ_3 being rational angles ("Ueber endliche Gruppen linearer Transformationen einer Veränderlichen," *Mathematische Annalen*, 1877, page 29). By forming the corresponding equation for a linear homogeneous group G in n variables it may be proved that all substitutions of such a group whose orders are products of primes greater than $(n-1)(2n+1)$ form a subgroup G_1. Professor Blichfeldt proved that G_1 is abelian and self-conjugate under G. He also pointed out the following results: (1) No simple group whose order is divisible by a prime greater than $(n-1)(2n+1)$ can be simply isomorphic with a linear homogeneous substitution group in n variables. (2) If the determinants of the substitution of a linear homogeneous substitution group in n variables are all unity, and if n is a prime, then the order of the group is not divisible by any prime greater than $(n-1)(2n+1)$.

4. This paper may be considered a continuation of the one published in the *Transactions*, volume 4 (1903), page 103. In it Professor Allardice gives an investigation of the loci of the vertices and foci of the system of conics through three points. He also considers the envelope of the directrices. Each pair of vertices and each pair of foci has a distinct locus, and each directrix has a distinct envelope. The loci are of the eighth and sixth degrees respectively, and the envelope is of the fourth degree.

5. Dr. Epsteen shows that a certain theorem of Lie's may be stated as follows: The necessary and sufficient conditions that a continuous r-parameter group G_r may contain an invariant subgroup g_s $(s < r)$, is that the adjoint group G_r shall be reducible. A number of consequences are easily deduced from this by utilizing the notions of reducibility of the adjoint group. For instance, if G_r is integrable, so is its adjoint group. If a group coincides with its adjoint, and contains an invariant subgroup, it is reducible. The quotient groups are the same for all composition series apart from their order. By introducing a notion, which in the theory of continuous group corresponds to the group of contragredient isomorphisms, the "total adjoint group" is defined, and the total adjoint group is then shown to be always reducible.

6. Several years ago Professor Tanner called attention * to the indications of the existence of a reciprocity theorem, viz., that in every group of totitives of order g there are as many subgroups of order g_1 as there are of order $g \div g_1$, g_1 being any divisor of g. As the groups of totitives are included among the abelian groups,† the theorem whose existence Professor Tanner suspected is included in the theorem that every abelian group of order g has as many distinct subgroups of order g_1 as there are subgroups of order $g \div g_1$. The last theorem is readily proved by means of the characteristics of an abelian group. Professor Miller proved this theorem without using characteristics and pointed out some of its applications.

7. At the first meeting of the Section, Dr. Moreno presented some results in regard to non-abelian groups in which every subgroup is abelian.‡ The present paper completes the determination of all such groups. The following are some of the most important results: The order of such a group cannot be divided by more than two distinct primes. If it is divisible by two primes (p, q), the Sylow subgroup whose order is a power of one of these primes is of type $(1, 1, 1, \cdots)$, while the other Sylow subgroup is cyclic. The former of these Sylow subgroups is also the commutator subgroup and includes no invariant operator besides the identity. When the order of such a group is p^a, it contains just $p + 1$ subgroups of order p^{a-1} and none of these can have more than three invariants. The paper will be offered to the *Transactions* for publication.

8. Mr. Manning investigated the primitive substitution groups of class $3p$, p being a prime, which contain substitutions of order p and degree $3p$. Jordan announced that the degree of a primitive group containg such a substitution cannot exceed $3p + 4$. By using this theorem it is shown that such groups do not exist when $p > 5$. When $p = 5$, there are the three primitive groups, G_{80}^{16}, G_{240}^{16}, and G_{4080}^{17}. The two cases, in which $p = 2$ and $p = 3$, are not here considered since they have already been worked out by Jordan.

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* Tanner, *Proc. London. Math. Soc.*, vol. 27 (1896), p. 329.

† *Annals of Math.*, vol. 2 (1900), p. 79.

‡ BULLETIN, vol. 8 (1902), p. 434. When P_{a-1} has only two invariants each of them may be unity. This special case was overlooked.

9. In a paper presented to the Society, October, 1901, Miss Schottenfels determined the generational definition of the abstract group of order 960 holoedrically isomorphic to the ternary group Galois field $[2^2]$ of the same order with the following definition:

$$(1)\qquad \begin{aligned} E_1^2 = E_2^2 = E_3^2 = B_1^2 = I,\\ (E_1E_2)^3 = (E_2E_3)^3 = (E_1E_3)^2 = I,\\ (B_1E_1) = (B_1E_2)^4 = (B_1E_3)^2 = I, \end{aligned}$$

where E_1, E_2, E_3, B_1 satisfying relations (1) are the generators of the group.

This group can be immediately extended by an operator C of order 7, say $C^7 = I$, to the abstract simple group of order $8!/2$ holoedrically isomorphic to the simple ternary group Galois field $[2^3]$ of order $8!/2$, with generators C, E_1, E_2, E_3, B_1, satisfying the further relations

$$(2)\qquad (CE_1)^5 = (CE_2)^5 = (CE_3)^5 = (CB_1)^5 = I.$$

10. In the determination of the cyclic subgroups of the quaternary group of determinant unity, taken in a general Galois field, the canonical forms with unequal multipliers give rise to some difficulties which do not present themselves in the others. Dr. Putnam investigates these cases, starting with the homogeneous group and passing from it to the fractional form in the usual way. The paper is supplementary to the article published in the *American Journal of Mathematics*, volume 24 (1902), page 319.

G. A. MILLER,
Secretary of the Section.

STANFORD UNIVERSITY.

A FUNDAMENTAL THEOREM WITH RESPECT TO TRANSITIVE SUBSTITUTION GROUPS.

BY PROFESSOR G. A. MILLER.

(Read before the American Mathematical Society, February 28, 1903.)

LET G represent any transitive group of degree n and of order $g = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_m^{\alpha_m}$; $p_1, p_2, \cdots, p_m$ being distinct primes. According to Sylow's theorem G contains at least one subgroup of each of the orders $p_1^{\alpha_1}, p_2^{\alpha_2}, \cdots, p_m^{\alpha_m}$. It will be convenient to speak of these subgroups as the *Sylow subgroups of* G. Since G is transitive, all the prime factors of n are included among the factors of g. The theorem in question may now be stated as follows:

THEOREM. *If p_1^β is the highest power of p_1 which divides n, each Sylow subgroup of order $p_1^{\alpha_1}$ in G has a transitive constituent of degree p_1^β and all its other transitive constituents are of degree $p_1^{\beta+\gamma}$, $\gamma \geqq 0$.*

COROLLARY I. *If $n = 2p^\alpha$, p being any odd prime, each of the Sylow subgroups whose order is a power of p has just two transitive constituents, each being of degree p^α.*

COROLLARY II. *If n is a power of a prime, a Sylow subgroup of G whose order is a power of the same prime must be transitive.*

The proof of the theorem results from the following elementary considerations: Let G_1 represent the subgroup of G which is composed of all its substitutions which omit a given letter. Every Sylow subgroup of G_1 is found in some Sylow subgroup of G. Whenever the orders of these subgroups are different the former is composed of all the substitutions of the latter which omit one letter.*

Since p_1 is any prime divisor of g we may confine our attention to the Sylow subgroups of G_1 and G whose orders are $p_1^{\alpha_1-\beta}$ and $p_1^{\alpha_1}$ respectively. At least one transitive constituent of the latter must be such that the order of its largest subgroups of lower degree than its own may be obtained by dividing its own order by p_1^β; *i. e.*, one of its transitive consti-

* Cf. Burnside, Theory of groups of finite order, 1897, p. 94.

tuents must be of degree p_1^β. If another transitive constituent were of degree p_1^δ, $\delta < \beta$, G_1 would contain a subgroup of order $p_1^{\alpha_1-\delta} > p_1^{\alpha_1-\beta}$. As this is impossible, a Sylow subgroup of order $p_1^{\alpha_1}$ contains at least one constituent of degree p_1^β but it contains no constituent of lower degree.

This proof remains true even if $\beta = 0$, for in this case a Sylow subgroup of order $p_1^{\alpha_1}$ is of degree less than n and hence may be supposed to have a constituent of degree 1. In all other cases the degree of this Sylow group is evidently equal to n. That the Sylow subgroup of order $p_1^{\alpha_1}$ may have constituents whose degrees exceed p_1^β follows directly from the three transitive groups of degree 6 and order 24. In two of these the Sylow subgroups of order 8 involve just two transitive constituents, of degrees 4, 2 respectively, while our theorem merely proves that there is a constituent of degree 2 and that there cannot be one of a lower degree.

As an application of this theorem we may prove the following: *If the degree of a simply transitive primitive group is* np^a, p *being any prime which does not divide* $n > 1$, *the maximal subgroup of degree* $np^a - 1$ *cannot involve a transitive constituent of degree* $(n-1)p^a$. If $a = 0$ this theorem requires no proof. If $a > 0$, the Sylow subgroup of order p^λ in G would contain two transitive constituents, of degrees $(n-1)p^a$, p^a, respectively. Hence G would involve a substitution not found in G_1 which would not connect all the transitive constituents of G_1. This is impossible since G_1 is maximal in a primitive group. For instance, in a simply transitive primitive group of degree 18, the maximal subgroup of degree 17 cannot involve a transitive constituent of degree 9.

STANFORD UNIVERSITY,
February, 1903.

THE CHARACTERIZATION OF COLLINEATIONS.

BY DR. EDWARD KASNER.

(Read before the American Mathematical Society, February 28, 1903.)

A COLLINEATION is ordinarily defined as a point transformation which converts collinear points into collinear points, *i. e.*, one for which the family of ∞^2 straight lines, or the equivalent differential equation $y''=0$, is invariant. The question suggests itself whether this definition does not contain redundancies—whether it is not sufficient to require that only some straight lines shall remain straight. If we understand by a *simple* system of lines any system possessing the property that through each point of the region of the plane considered there passes one and only one line, then the result of this note may be stated:

If four simple systems of straight lines remain straight after a point transformation, then the same is necessarily true of all straight lines, and the transformation is, therefore, a collineation.

To prove this consider the general point transformation T

$$X=\phi(x,y), \quad Y=\psi(x,y), \tag{1}$$

where ϕ and ψ are one-valued continuous functions possessing first and second derivatives in the region considered. It is assumed, of course, that the Jacobian, $J=\phi_x\psi_y-\psi_x\phi_y$, does not vanish identically. The once extended transformation is

$$Y'=\frac{\psi_x+\psi_y y'}{\phi_x+\phi_y y'}; \tag{2}$$

and the twice extended may be written *

$$Y''=\frac{a+\beta y'+\gamma y'^2+\delta y'^3+Jy''}{(\phi_x+\phi_y y'^3)}, \tag{3}$$

where

$$\begin{aligned} a&=\phi_x\psi_{xx}-\psi_x\phi_{xx},\\ \beta&=\phi_y\psi_{xx}-\psi_y\phi_{xx}+2\phi_x\psi_{xy}-2\psi_x\phi_{xy},\\ \gamma&=\phi_x\psi_{yy}-\psi_x\phi_{yy}+2\phi_y\psi_{xy}-2\psi_y\phi_{xy},\\ \delta&=\phi_y\psi_{yy}-\psi_y\phi_{yy}. \end{aligned} \tag{4}$$

* Lie-Scheffers, Continuirliche Gruppen, 1893, p. 33.

If now T transforms any straight line into a straight line, then for that line $y'' = 0$ and $Y'' = 0$, so that from (3)

$$\alpha + \beta y' + \gamma y'^2 + \delta y'^3 = 0. \tag{5}$$

This differential equation of the first order and third degree is satisfied by all the invariant straight lines. Assuming that there are four simple systems of such lines, then through each point there pass four; these give four values of y', corresponding to fixed values of $\alpha, \beta, \gamma, \delta$. Equation (5) is therefore an identity. This means, however, that $Y'' = 0$ is a consequence of $y'' = 0$, or that $y'' = 0$ is invariant under T.* This proves the result stated above.

The result may be restated in terms of differential equations as follows. A simple infinity of straight lines is represented by a Clairaut differential equation

$$y = xy' + f(y'), \tag{6}$$

where y' is a one-valued function of x, y. If an equation of this kind is transformed into a similar equation, then the corresponding lines remain straight. Hence, *if T transforms four Clairaut equations of the first degree into Clairaut equations, then the same is true of all Clairaut equations, and T is a collineation.*

It may be added that what has been obtained is, in a sense, *the minimum definitive property of the collineations.* For there exist non-projective transformations which possess one, two, or three simple systems of invariant straight lines. A familiar example of the latter type is given by inversion with respect to a circle; here the three invariant systems are the pencils through the circular points at infinity and through the center of the circle. An example where all three systems are real is given by the quadratic transformation $X = 1/x$, $Y = 1/y$, which leaves invariant the systems $x = \text{const.}$, $y = \text{const.}$, $y/x = \text{const.}$ The same systems are invariant under the three parameter group $X = ax^c$, $Y = bx^c$.

The problem of determining all transformations possessing three invariant systems of straight lines is one of considerable difficulty. The writer expects to discuss it, as well as the extension of the results to space, in a future paper.

COLUMBIA UNIVERSITY.

* The direct integration of the equations $\alpha = \beta = \gamma = \delta = 0$, leading to collineations, was accomplished by Scheffers; cf. p. 34, l. c. For infinitesimal transformations the analysis is simpler. Lie-Scheffers, Differentialgleichungen, 1891, p. 389.

A MODERN FRENCH CALCULUS.

Cours d'Analyse Mathématique. Par ÉDOUARD GOURSAT, Professeur à la Faculté des Sciences de Paris. Tome I. Gauthier-Villars, Paris, 1902.

THE revision of the fundamental principles of the calculus, which was initiated by Cauchy and Abel and carried through by Weierstrass and his followers, led to the development of the ϵ-proof (early introduced by Cauchy) and to the precise formulation of definitions and theorems. In Germany and Italy a tendency sprang up to place only such restrictions on definitions and theorems as are necessarily imposed by the nature of the case. Thus functions continuous throughout no interval whatever were admitted as the integrand of a definite integral simply because the form of the definition of the integral applied to a certain class of these functions, and the question was examined of how far the ordinary theorems of the integral calculus hold for such functions. Again, the theorem that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ was proved with fewer restrictions than the continuity of all the derivatives that enter. While this procedure is perfectly justifiable so far as it is a question of research in a special field, it is important not to lose sight of the fact that investigations of this sort are but a very special phase of modern analysis, and that even the specialist in the field of analysis may never need to trouble himself about the integrals of other functions than those which are continuous except at a finite number of points. That which is essential for every mathematician to know who has occasion to use the calculus to any extent is a simple formulation of the theorems and simple tests for the validity of the processes of the calculus which have been handed down to us from Euler's time and earlier: — when may a convergent series of continuous functions be integrated term by term, when may a definite integral whose integrand satisfies reasonable conditions of continuity be differentiated under the sign of integration? These are questions of general interest to mathematicians. To the importance of a simple and lucid answer French mathematicians are alive. With full appreciation of modern standards of rigor they do

not allow themselves to obscure in their presentation of the calculus the main facts of analysis by cumbersome details.

The book before us belongs to the best type of modern French treatises on the calculus. It is based on Professor Goursat's university lectures. According to the plan of instruction in France the student of mathematics learns at the *lycée* the meaning and use of derivatives, differentials not being introduced, and the rudiments of algebraic analysis.* Thus the university teacher can assume that the student is familiar with the notion of the limit and the elementary methods of the calculus, and that he has sufficient maturity to understand a treatment of the calculus such as is given in American universities in a second and third course.

After some introductory paragraphs, in which Rolle's theorem and the law of the mean are given, the author proceeds to a systematic treatment of partial differentiation, based on the total differential of a function. The fundamental theorems on which the properties of such differentials rest and which are overlooked in English text-books on the calculus† are here given their proper place.‡

Chapter II, pages 40–100, begins with the existence theorem for implicit functions, the proof being that which Dini gave in his lectures of 1877/78; and then follows the differentiation of the functions thus defined, properties of the Jacobian determinant, and change of variable. In algebra and algebraic analytic geometry the mere rudiments of partial differentiation suffice for the applications that arise; but in differential geometry and mathematical physics this is not the case. It is highly desirable that partial differentiation should be studied more at length than is at present the case, and a complete and lucid treatment such as is here given § will aid the teacher in modifying his course in that direction.

In American colleges students of calculus are not mature enough to appreciate existence theorems at the time when they

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* *Cf.* Pierpont, "Mathematical instruction in France," BULLETIN (2), 6 (1900), p. 225.

† *Cf.* The writer's review: "A modern English calculus," BULLETIN (2), 8 (1902), p. 253.

‡ *Cf.* § 16, top of p. 29, and the corresponding theorem in § 14.

§ The present treatment is much fuller than that of Jordan, Cours d'analyse, and is illustrated by numerous applications of substantial character. A few examples taken from thermodynamics would have been a useful addition.

study partial differentiation and it is best to assume that equations defining implicit functions can be solved and lead to functions which have derivatives. It is easy, however, in using Professor Goursat's book, to assume these theorems at this stage and take them up later. In a logical development of the calculus they belong where the author has put them and for the working mathematician the arrangement here adopted is the most satisfactory one.

A few matters of detail before leaving these chapters. On page 5, line 12 from the top, the words "et à rester" should be inserted before "supérieure." The definition of approaching a limit in the case of a function of several variables is not given; the continuity of such a function is however carefully defined. The law of the mean for functions of several variables might well have been given a place in Chapter I. It appears nowhere explicitly, merely as a special case of Taylor's theorem with the remainder in Chapter III. On page 45, near the bottom, it is not of course true that if the three partial derivatives vanish simultaneously, the point is necessarily a singular point. This oversight occurs at various other places in the book. In the theorem at the beginning of § 28 it is necessary for the proof that for the system of values of $(u_1, \cdots, u_n)$ considered the partial derivatives $\Pi_{u_1}, \cdots, \Pi_{u_n}$ should not vanish simultaneously. This requirement should be made in the statement of the theorem. In this section (§ 28) the author does not bring out with all the clearness one could wish the fact that there are in all three cases: (*a*) the case in which the Jacobian determinant does not vanish; (*b*) the case in which it vanishes identically; and (*c*) the case in which it vanishes at a given point, but not at all points in the neighborhood. Cases (*a*) and (*b*) are treated. But little is known about case (*c*), and there is no reason for considering it; but the classification is important. A large number of examples are given at the end of Chapters I and II, all of which are taken from life. While the student will need to do many easy examples, like those given in English books on calculus, at the start, he will have the satisfaction in working these more difficult ones of knowing that they are not artificial, that they truly illustrate the actual applications of partial differentiation.

Chapter III is devoted to Taylor's theorem with the remainder and to Taylor's series, both for functions of one and for functions of several variables, and to applications. The

French have not committed themselves to the exclusive use of power series.* When it is simpler to use Taylor's theorem with the remainder, they do so; and it is an excellent feature of the book before us that the infinite series is not employed when the finite series is better suited to the purpose. Indeterminate forms are treated by the use of Taylor's theorem, but we miss the more general theorem that if $f(a) = \phi(a) = 0$ or ∞, and if $f'(x)/\phi'(x)$ approaches a limit, then $f(x)/\phi(x)$ approaches a limit and $\lim f(x)/\phi(x) = \lim f'(x)/\phi'(x)$. Thus such limits as

$$\lim_{x=0} x \log x \qquad \text{or} \qquad \lim_{x=\infty} \frac{\log x}{x},$$

or more generally

$$\lim x^{\alpha} (\log x)^{\beta},$$

when $x = 0$ or ∞; or again

$$\lim_{x=\infty} x^{n} e^{-x}$$

must each be evaluated by a special investigation,—one for which no general method is given.

The treatment of maxima and minima of functions of several variables is admirable, adequate and clear, but not overdone.† The examples studied are well chosen and include clothed problems that do not come under the ordinary rule.

Chapters IV–VII, pages 150–368, are entitled respectively Definite Integrals, Indefinite Integrals, Double Integrals, and Multiple Integrals, Integration of Total Differentials. In American colleges the custom is growing of introducing the definite integral, defined as the limit of a sum and represented by the area under a curve, early in the first course in calculus, and of applying it to the solution of a variety of problems in physics and mechanics. Thus the student comes to the second course in calculus with a pretty good idea of what integrals are

* This tendency in Germany is well hit off by Bohlmann in his report on important works on the calculus, *Jahresbericht der Deutschen Mathematiker-Vereinigung*, vol. 6 (1897), p. 110. One of the entries in the table of contents reads "Es gibt nur Potenzreihen!"

† The definition of a maximum or a minimum on page 130 would seem to exclude the case that $f(c+h) - f(c)$ can vanish for other values of h than the value $h = 0$; but this is not the definition employed on page 136: "Le cas où $b^2 - ac = 0 \cdots$."

and what they are for, and he is prepared for the study of double integrals, improper integrals and functions defined by integrals. It is at this stage, that is, in the second calculus course in American colleges, that these chapters of Professor Goursat's book may be employed with great advantage. They give a simple and rigorous elementary treatment of the subjects just mentioned — particularly the chapter on double integrals, pages 282–333 — a masterpiece of presentation, so clear and rigorous and to the point that it may well serve as a text in the treatment of this subject. Regarding the first of these chapters a similar remark applies to the one made concerning the existence theorem for implicit functions in Chapter II; namely, that in the second course in calculus it is well to assume the theorems about continuity, the proof coming more properly at a later stage, when questions of uniform convergence are treated; but here again the arrangement which the author has adopted will commend itself to the working mathematician as being the natural sequence of topics. There is one omission that seems to us unfortunate. It is that of Duhamel's theorem regarding the representation of any infinitesimal in a limit of a sum by another infinitesimal that differs from it by one of higher order. But as we are treating this subject at length elsewhere, we restrict ourselves here to a reference.*

The following topics considered in these chapters deserve special mention because of the lucid and rigorous treatment: change of variables in simple and multiple integrals §§ 84, 128,† 130, 145, 150; improper integrals simple and multiple,

* *Annals of Math.* (2), 4 (July, 1903).

† The method here set forth of proving by means of Green's theorem that when curvilinear coördinates $x = f(u, v)$, $y = \phi(u, v)$ are introduced, the area Ω bounded by the curves $u = u_0$, $u = u_0 + du$, $v = v_0$, $v = v_0 + dv$ is

$$\Omega = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} du\, dv,$$

where the Jacobian is to be formed for a certain point (u', v') in Ω, is especially elegant. The author's reason for not extending it to triple integrals was doubtless that the formulas become a little long; but they present no serious difficulty and the generalization may well serve as an exercise for the student, his attention being first called to the fact that the three-rowed Jacobian may be written in the form

§ 89*–91, 133; line and surface integrals, including Green's and Stokes's theorems, §§ 93, 126, 135, 136, 149, 151–155: differentiation under the sign of integration, § 97; approximate calculation of an integral by Simpson's and Gauss's rules; Amslers's planimeter, § 99–102; the geometric interpretation of the method of rationalization employed for computing the integrals $\int R(x, \sqrt{a+bx+cx^2})dx$, § 105.

Chapters VIII, IX, pages 369–478, deal with infinite series. The subject is treated *ab initio* and the ordinary tests for convergence and theorems relating to the algebraic transformation of series are developed with the clearness and rigor that mark the whole book. But the last page of § 169 relating to double series whose terms are not all of like sign needs to be expanded. — The definition of uniform convergence in the case of infinite series is the one given by Darboux,† and is as follows: The series of continuous functions

$$u_0(x) + u_1(x) + \cdots,$$

convergent in the interval (a, b), is said to be uniformly convergent in this interval if to every positive ϵ there corresponds a positive integer n, independent of x, such that the remainder

$$R_n(x) = u_n(x) + u_{n+1}(x) + \cdots,$$

remains in absolute value less than ϵ for all values of x in the

$$\begin{vmatrix} f_u & f_v & f_w \\ \phi_u & \phi_v & \phi_w \\ \psi_u & \psi_v & \psi_w \end{vmatrix} = \frac{\partial}{\partial u}\begin{vmatrix} f_v & f_w \\ \phi_v & \phi_w \end{vmatrix} + \frac{\partial}{\partial v}\begin{vmatrix} f_w & f_u \\ \phi_w & \phi_u \end{vmatrix} + \frac{\partial}{\partial w}\begin{vmatrix} f_u & f_v \\ \phi_u & \phi_v \end{vmatrix}.$$

The possibility of this extension was pointed out by the author in his original paper, *Bulletin des sciences math.* (2), 18 (1894), p. 72.

* The theoretical developments here given are especially simple. The rule for the convergence of the integral

$$\int_a^b f(x)dx,$$

when $f(x)$ is discontinuous at the point $x = a$, page 198, bottom, can, however, be formulated still more simply if we require merely that for some value of $0 < k < 1$ the variable $(x-a)^k f(x)$ shall approach a limit when x approaches a. A similar remark applies to the formulation of the other tests for convergence and divergence of improper integrals.

† *Ann. École norm.* (2), 4 (1875), page 77.

interval. It will be observed that this definition differs from the one ordinarily given in that it does not require that $|R_n(x)| < \epsilon$ for *every* $n > m$, but only for *one* value of n. Nevertheless the proof that such a series represents a continuous function is sound. The proof that the series may be integrated term by term is, however, invalid, as the following example shows: let

$$S_n(x) = nxe^{-nx^2} \text{ when } n \text{ is odd};$$
$$S_n(x) = 0 \quad \text{“} \quad n \text{ “ even.}$$

Here the term by term integral in the interval (0, 1) has the value

$$\int_0^1 S_n(x)dx = \tfrac{1}{2}(1 - e^{-n}) \text{ when } n \text{ is odd};$$
$$= 0 \quad \text{“} \quad n \text{ “ even.}$$

What is true is this: if such a series be integrated term by term and parentheses then be suitably introduced, the series of parentheses will converge toward the integral of the function as its limit. Tannery * has called attention to the fact here mentioned, namely, that a series uniformly convergent according to Darboux's definition can not always be integrated term by term, but both Picard † and Goursat appear to have overlooked Tannery's corrections. In § 175, however, which deals with the differentiation of an improper integral

$$\int_a^\infty f(x, \alpha)dx$$

under the sign of integration, the definition of the uniform convergence of such an integral is the one ordinarily given. If such an integral converges uniformly (and the integrand is continuous in x, α), it represents a continuous function of α and may be integrated under the sign of integration:

$$\int_{\alpha_0}^{\alpha_1} d\alpha \int_a^\infty f(x, \alpha)dx = \int_a^\infty dx \int_{\alpha_0}^{\alpha_1} f(x, \alpha)d\alpha.$$

These theorems are precisely analogous to the theorems about

* Fonctions d'une variable, Paris, 1886, p. 366, foot-note. The foot-note of pages 133–4, so far as it relates to integration, contains an inaccuracy, which is corrected in the later foot-note just cited.

† Traité d'analyse, vol. I, chap. VIII, 1st ed., p. 195; 2d ed., p. 211.

uniformly convergent series and should be given along with the sufficient condition the author deduces for differentiating under the sign of integration. The theorems relating to the transformation of power series in one and in several variables, and the theorems that follow from these, together with the proof of the analytic character of the implicit functions defined by a system of analytic equations, $F_i(x_1, \cdots, x_q; y_1, \cdots, y_p) = 0$ $(i = 1, \cdots, p)$ are all faithfully developed, pages 419–461 being devoted to this topic. The method of the *fonctions majorantes* is explicitly set forth. Analytic functions of real arguments are defined by the property that they admit a development by Taylor's theorem. The author does not neglect to remind the reader that in spite of the important rôle that these functions play, they form after all but a very special group of real functions of real variables within the general group of continuous functions. Finally the development of a continuous function into a Fourier's series is established. The volume ends with three chapters, pages 479–610, on applications of the differential calculus to curves and surfaces.

The extraordinarily high standard of simplicity and attractiveness in style, combined with modern rigor, which Picard set in his Traité d'analyse is fully maintained by Professor Goursat. The objects of the two works are quite different. Picard's purpose was to write a treatise on differential equations, and he developed only such parts of analysis and geometry as bear on this subject. Goursat, on the other hand, has set himself the task of writing a systematic treatment of the calculus, and thus the whole field of the calculus is included here.

A treatise on advanced calculus which should present the whole subject rigorously and attractively, and at the same time in the spirit of modern analysis, has been sorely needed by students of mathematics who intend to proceed to the study of mathematical physics or of some of the various branches of analysis — theory of functions, differential equations, calculus of variations, etc. Professor Goursat's work meets the needs of such students in a thoroughly satisfactory manner, and we recommend it to them most heartily. The teacher of calculus will find many suggestions in the book which will enable him to improve his course, and he may often with advantage refer even an elementary class to the more elementary parts of the book for collateral reading. The range of the book is wide.

While beginning with the elements of the calculus, it carries the reader to the point where he is prepared to use original sources and extracts from ϵ-proofs the underlying thought. When the future historian inquires how the calculus appeared to the mathematicians of the close of the nineteenth century, he may safely take Professor Goursat's book as an exponent of that which is central in the calculus conceptions and methods of this age.

W. F. OSGOOD.

HARVARD UNIVERSITY,
February, 1903.

SHORTER NOTICES.

Niedere Zahlentheorie. Erster Teil. By Dr. PAUL BACHMANN. B. G. Teubner's Sammlung von Lehrbüchern auf dem Gebiete der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen. Band X, 1. Leipzig, 1902. x + 402 pp.

IN view of the ambitious series of volumes by Bachmann, giving a comprehensive exposition of number theory, a series not yet completed, the appearance of a new volume on the elements of the subject, quite independent of the series mentioned, will doubtless cause some surprise. When the invitation came to the author to contribute to Teubner's Sammlung a text upon the subject on which he is so eminent an authority, he hesitated long, fearing that a text on the elements of number theory ran the risk of conflicting with his Elementen. The author has attempted to avoid this conflict in two directions: first by the addition of much important material; second, by employing a method of construction different at least in essential points. The author believes that the present book, both in contents and in foundation, may well be considered as a supplementary volume to his former series. As indicating in detail parts differing essentially from the Elementen, there may be mentioned the chapter on the different euclidean algorithms, including Farey's series, the theory of binomial and general congruences, the exhaustive treatment of the known proofs by elementary number theory of the quadratic reciprocity law and the interrelations of these proofs. The theory of higher congruences is appropriately introduced, even in the Niedere Zahlentheorie, both by way of climax to the elementary parts and to afford a satisfactory insight into the means employed by Gauss in his seventh proof of the reciprocity law.

The introduction (15 pages) gives a brief history of number theory. Chapter I is a rather formal discussion of the concept number, the idea of sequence being taken for the foundation. The 38 pages of Chapter II are devoted to the divisibility of numbers. On pages 34 and 35 is given a simple proof of the existence of integers ξ and η satisfying the equation $a\xi + b\eta = \delta$, where δ is the greatest common divisor of a and b, use being made of the *modulus* of Dedekind and Kronecker. But this proof of the fundamental theorem gives no algorithm for finding ξ and η as does the proof based on Euclid's algorithm. A series of special recent theorems due to Weill, André, de Polignac, Catalan, Landau, and Liouville show that certain expressions in fractional form have integral values. The 33 pages of Chapter III are devoted to the usual theorems on residues and congruences. The 54 pages of Chapter IV are devoted to the euclidean algorithm, continued fractions, Farey's series. The 27 pages of Chapter V relate to the theorems of Fermat and Wilson and their generalization. Chapter VI, on the theory of quadratic residues, is the longest in the book, containing 138 pages; it is devoted chiefly to the various proofs of the reciprocity law, a chronological table of which appears on pages 203 and 204, with an addition on page 402. The 82 pages of the final Chapter VII gives the theory of higher congruences. The theory of primitive roots is treated at length with two distinct proofs of the existence of primitive roots modulo p, a prime. A general theory, including that of circulating decimals, is then developed. Finally follows the chief results of Galois, Schönemann, Dedekind, and Serret on higher congruences. On pages 373–375 is given Gauss's elegant and simple method of determining the number of incongruent primary irreducible functions (mod p) of degree n; following this is the usual purely arithmetic development.

The author has certainly succeeded admirably in the task he has undertaken. It will appeal especially to the student who wishes a complete account of the results and methods of the elementary parts of the theory of numbers.

L. E. Dickson.

The Constructive Development of Group Theory, with a Bibliography. By BURTON SCOTT EASTON. Philadelphia, Publications of the University of Pennsylvania. Mathematics, No. 2, 1902. 8vo, iv + 89 pp. Ginn & Co., Selling Agents.

"THE purpose of this monograph is to present in a consecutive form the principal features of abstract and substitution group theory." It has been the aim of the author "to examine in detail all memoirs dealing directly with such group theory (excluding, in particular, that of linear groups) and to construct from this material a continuous treatise on the subject." These few words from the preface are sufficient evidence of the importance and difficulty of the task which the author has set for himself.

The monograph consists of two parts — a bibliography, covering pages 5–38; and a compendium, covering pages 39–86. The former is preceded by a prefatory note and a list of periodicals with abbreviations, while the latter is followed by a good subject index. The bibliography includes 157 names and gives under each name the titles and places of publication, arranged in chronological order. As this is the first publication of its kind, it is a very welcome addition to the literature on the subject and will doubtless prove valuable to all workers in this field.

Although great care seems to have been exercised in compiling the bibliography, yet it can scarcely be expected that the first edition of such a comprehensive collection would be free from errors. A few corrections are made in the addenda which closes the memoir. To these should be added Tanner's paper "On the enumeration of groups of totitives," *Proceedings of the London Mathematical Society*, volume 27; Hurwitz, "Algebraische Gebilde mit eindeutigen Transformationen in sich," *Mathematische Annalen*, volume 41; Study, "Sphärische Trigonometrie," *Leipziger Abhandlungen*, volume 20; the algebras of Comberousse and Pincherle, and possibly one or two of the papers of H. Laurent.

Many readers will doubtless regret that the author did not aim to include all papers on linear groups. This subject is so closely related to that of abstract and substitution groups that it frequently seems difficult to decide whether a paper on one of these subjects has sufficient bearing on the other to be classed with its literature. Such an extension would however have made the task of the author still more difficult, and he

can scarcely be blamed for leaving this part to some one else. We may note here a slight typographical error. Under M 24 the word "where" should be replaced by *whose*.

The second part practically consists of the statement of theorems without proof and a comprehensive list of references to works where proofs may be found. These references are arranged in chronological order and tend to make this part even more useful than the first. By employing abbreviations very freely a vast amount of information is crowded on less than fifty pages. It is to be hoped that many of the readers will have the kindness to comply with the author's request and notify him of any inaccuracies which they may observe. The following have been observed by the reviewer:

In no. 134 it should be stated that g is a power of a prime. Otherwise the theorem is not generally true. With this change the theorem belongs under heading 9, "Groups whose order is a power of a prime number." As a special case of this theorem no. 119 should read $m>6$ instead of "$m>p+4$." Of course, the latter is true, but it is not as general as the former. In no. 142 the term "subgroup" at the end of the first line should be replaced by *group*, and in no. 146 the expression "two abelian subgroups" should be replaced by *an abelian group*. In no. 166 for $n-1$ read $m-1$, and replace "primitive group" by *primitive perfect group* in 168.

In nos. 171–173 it should be stated that the primitive groups in question do not include the alternating group and *greater than* 3 should be added to the first line of no. 175. A curious oversight is presented by no. 219. The holomorph of G is never the direct product of G and its i-group. In no. 20 the exponents should evidently be $m_1, m_2, \cdots$ in place of $m1, m2, \cdots$

According to Cantor's dictum * "da die Geschichte unwiderruflich die Veröffentlichungszeit als allein massgebend betrachten muss, wo Erstlingsrechte zu vergeben sind" it would appear that the expression "Due to Dd" under no. 103 should either be omitted or replaced by *Due to M.*† The references under no. 105 are somewhat misleading as the theorem was first given in this form by M 13,267. Its proof was directly based upon older theorems and due acknowledgment was made in the first publication.

G. A. MILLER.

* Cantor, Vorlesungen über Geschichte der Mathematik, vol. 2 (1900), p. 811.

† Frobenius, *Berliner Sitzungsberichte* (1896), p. 1348.

Enumération des Groupes d'Operations d'Ordre Donné. Par RAYMOND LE VAVASSEUR, à Toulouse. Paris, Hermann. 4to, pp. 128, lithographed. No index or title-page. Fr. 5.00.

THE appearance of a new treatise on groups has ceased to be an event in the mathematical world, but M. Le Vavasseur's work is something a little out of the ordinary. For by means of nine chapters devoted respectively to groups of orders p, p^2, pq, 8, $pq^2(p > q)$, 16, p^2q, $8p$, p^3 and pqr its purpose is to introduce persons unfamiliar with group theory directly to the problems of group construction. And hence everything in each of these chapters is directed towards answering the question, "How many groups are there of the assigned order and what are their equations?"

No knowledge whatever of the subject is assumed. The author begins with the fundamental definitions of the theory and develops the necessary preliminary propositions in a manner that follows Burnside's treatment with considerable exactness, introducing, for instance, into the French terminology such phrases as suite complète, opération conjuguée d'elle même. The whole of the second chapter is adapted from Burnside—so completely that the writer might have expressed an acknowledgment of his source more fully than in the single reference on page 16. These chapters and those immediately following are devoted to the simpler cases of group construction, too familiar ground to admit of much originality in treatment, but the discussion given is lucid and tolerably concise. It is certainly less prolix than Hölder's standard paper in volume 43 of the *Mathematische Annalen*. But this virtue of brevity cannot be claimed for the analysis of groups of order $8p$, as this chapter fills sixty-four pages and embodies a treatise on the group of automorphisms of abelian groups of order p^3 and type (1, 1, 1), with the representation of such groups in linear form and an excursus on bilinear substitutions. While the developments are interesting and while the results are perfectly correct (albeit the conclusions for order 56 are not very clear), yet the results could be attained in a very small fraction of the space used—cf., *e. g.*, Professor Miller's paper in volume 43 of the *Philosophical Magazine*.

All orders less than 30 are discussed with complete thoroughness, and the groups for each case are determined and their equations given. A comparison with Hölder's results seems to

show a simpler form of the defining relations, but inasmuch as the simplification is gained by using generators of a higher order its advantage is more or less a matter of taste. The arrangement of the tables is good, but the number of groups for a particular order is not always quite obvious.

The chief defect of the book, however, is its plan. It is evidently intended for beginners, but to set beginners immediately at work on group types is a procedure of very dubious wisdom. The methods involved are necessarily mechanical and monotonous—monotonous in that they demand the solution of a large number of congruences that differ but slightly, and mechanical in that the solution of such congruences adds little to the mathematical knowledge of the student. Moreover, the particular methods employed do not seem well adapted for extension to groups of a higher order—if the chapter on order $8p$ were a fair criterion, more complex groups would necessitate a prohibitive amount of labor. The last chapter however — on orders free from a quadratic factor—adopts Frobenius's method of a congruence with a double modulus, but terminates so abruptly that the author evidently intends to complete it later.

The book is unusually easy to read. Every needed theorem from other branches of mathematics is always fully stated and generally proved. And to those who are studying relations between abstract and linear groups the book may be recommended without hesitation.

B. S. EASTON.

Elemente der Theorie der Determinanten mit vielen Uebungsaufgaben. Von Dr. P. MANSION. Dritte vermehrte Auflage. Leipzig, Teubner, 1899. 103 pp.

THIS book, which has passed through six French editions, is an excellent introduction to the more extended treatises of Baltzer, Pascal, and Gordan. Its scope is distinctly elementary. The more complicated developments of determinants, as Laplace's, are passed over with a statement of fact without proof while functional determinants, and the general development with respect to a row and column simultaneously are not mentioned.

The book opens with an introduction of twenty-two pages on determinants of the second and third orders, which are illustrated with a large number of problems. The usual discussion of the elementary properties of the general determinant follows. The points of greatest pedagogical interest are, first, the admir-

ably clear and thorough account of the special cases that occur in the solution of linear systems and their dependence, second, the treatment of Sylvester's dialytic method of elimination in which not only the necessity but the sufficiency of the condition that two equations have λ common roots is derived — a point in which most elementary texts are painfully deficient. A brief and hence necessarily unsatisfactory note appears in the appendix on the development of the properties of determinants by means of alternate numbers, a method which is extremely rapid, and with somewhat mature students quite teachable.

No geometrical applications are given, except in isolated examples. Problems are abundant, well graded and not trivial. Not only is the book sufficiently elementary to warrant its wide use as a text-book, but its many pedagogical excellencies make it very valuable to a teacher of the subject.

H. E. HAWKES.

Liniengeometrie mit Anwendungen. Band I. By Dr. KONRAD ZINDLER, Professor an der Universität Innsbruck. Sammlung Schubert, XXXIV. Leipzig, G. J. Göschen, 1902. 8vo., viii + 380 pp., 87 figures.

IN this systematic exposition of line geometry, both synthetic and analytic methods are employed. It is the first book of this character since the appearance of Plücker's original treatise.* In fact, Sturm's Liniengeometrie in three volumes (1892, 1893, 1896) is purely synthetic; while other books, such as Koenigs's Géométrie réglée, treat only special parts of the subject.

Line geometry treats of manifoldnesses of straight lines in space. The names ruled surface, congruence, complex are applied to a one-fold, two-fold, three-fold system, respectively.

The present (first) volume treats chiefly of linear complexes and congruences and their applications; the second volume will be concerned chiefly with algebraic line configurations of higher than the first degree. The book is intended as a text-book; eighty-two exercises are scattered through the book, hints for the solution of certain of which are given in the appendix, with references for the others.

Chapter I treats of the Nullsystem and the Strahlengewinde. Let all the points of space be subjected to a given screw motion

* Neue Geometrie des Raumes, gegründet auf die Betrachtung der geraden Linie als Raumelement, I, 1868; II, 1869.

and let each point be made to correspond to that plane through it which is normal to the path of the motion. Inversely, to every plane corresponds a point. This one-to-one geometric correspondence is called a Nullsystem. If a point describes a straight line, the corresponding plane turns about another line, and inversely. Moreover, the correspondence preserves incidences. Hence a Nullsystem is a correlation in the sense of projective geometry. The totality of normals, at all points of space, to the path of a screw motion forms a Strahlengewinde; it is composed of ∞^3 straight lines; every point of space and every plane of space is the bearer of a flat-pencil of these lines. A simple visualization of the lines of a Strahlengewinde may be obtained by taking certain ∞^2 tangents to each of ∞^1 coaxial right circular cylinders.

Chapter II treats at length (pages 31–71) of applications to the theory of motion, mechanics and graphic statics. Chapter III treats in fifty-five pages the subject of line coördinates and the coördinates of a segment (Stab) defined by the length and the position of the line on which the sect lies. By way of introduction, the chief properties of homogeneous point and plane coördinates are given without proof.

Chapter IV treats of linear configurations of lines with applications to mechanics. A linear homogeneous equation between the six segment coördinates defines a linear complex. It is shown to be identical either with a Strahlengewinde or else with the totality of lines intersecting a given one. Hence the linear complex can be represented by a linear homogeneous equation in line coördinates. Its coefficients define a correlation in space for which every point lies in its corresponding plane; inversely, every such correlation may be defined by a Strahlengewinde. The section of two linear complexes is a congruence of the first order and first class, called (after Sturm) a net of lines. The properties of nets, including a complete classification, are given on pages 160–197.

Chapter V (pages 203–278) treats of imaginary elements, and their geometric interpretation, including a historical and logical discussion. The final chapter, VI, relates to manifoldnesses of linear complexes and applications to mechanics and the theory of motion. Among the topics treated may be mentioned the cylindroid (a ruled surface of the third order), general complex coördinates, and Klein line coördinates.

L. E. Dickson.

NOTES.

THE July number (volume 4, number 3) of the *Transactions* of the AMERICAN MATHEMATICAL SOCIETY contains the following papers: "On the point-line as element of space: a study of the corresponding bilinear connex," by E. KASNER; "On the formation of the derivatives of the lunar coördinates with respect to the elements," by E. W. BROWN; "On reducible groups," by S. EPSTEEN; "Theory of linear associative algebra," by J. B. SHAW; "Projective coördinates," by F. MORLEY; "On an extension of the 1894 memoir of Stieltjes," by E. B. VAN VLECK; "On the variation of the arbitrary and given constants in dynamical equations," by E. W. BROWN; "The primitive groups of class $2p$ which contain a substitution of order p and degree $2p$," by W. A. MANNING; "Complete sets of postulates for the theory of real quantities," by E. V. HUNTINGTON.

THE July number (volume 25, number 3) of the *American Journal of Mathematics* contains: "Isothermal-conjugate systems of lines on surfaces," by L. P. EISENHART; "Some differential equations connected with hypersurfaces," by G. O. JAMES; "On the forms of sextic scrolls of genus greater than one," by VIRGIL SNYDER; "Geometry on the cuspidal cubic cone," by F. C. FERRY.

THE concluding (July) number of volume 4 of the *Annals of Mathematics* contains: "On the characteristics of differential equations, Part II," by E. R. HEDRICK; "On the uniformity of the convergence of certain absolutely convergent series," by M. BÔCHER; "The integral as the limit of a sum, and a theorem of Duhamel," by W. F. OSGOOD; "On a general relation of continued fractions," by R. E. MORITZ; "Note on the equilateral hyperbola," by J. A. VAN GROOS; "A new proof of the generalized Wilson's theorem," by G. A. MILLER; "A sufficient condition for the maximum number of imaginary roots of an equation of the nth degree," by E. B. VAN VLECK.

AT the Fourth Colloquium, to be held in connection with the coming summer meeting of the AMERICAN MATHEMATICAL

SOCIETY (cf. previous announcement, BULLETIN, current volume, page 384) Professor H. S. WHITE will give a course of three lectures on "Linear systems of curves on algebraic surfaces." The Colloquium will open on Wednesday morning, September 2, and close on the following Saturday morning. The twelve lectures given in the three courses will be distributed as follows: two on each morning, Wednesday to Saturday inclusive, and two on each of two afternoons. The usual charge of three dollars will be made to those attending any or all the courses.

AT the meeting of the London mathematical society held on April 16, 1903, the following papers were read: By Mr. C. S. JACKSON: "Exhibition of the logo-logarithmic slide rule"; by Mr. R. F. GWYTHER: "On the deduction of Schlömilch's series from a Fourier series, and its development into a definite integral"; by Mr. E. T. WHITTAKER: "On those functions which are defined by definite integrals with not more than two singularities"; by Professor A. E. H. LOVE: "Note on exact solutions of the problem of the bending of an elastic plate under pressure"; by Professor A. LODGE: "Relations between points (in a plane) having conjugate complex coördinates."

At the meeting held on May 14, the following papers were read: By Dr. H. F. BAKER: (1) On the definiteness of quadratic forms with imaginary coefficients; (2) On a certain form of logical argument which occurs in the proofs of several fundamental theorems of pure mathematics; (3) On the summation of Neumann's series representing a potential defined by boundary values; (4) On the formation of the variant equation in the theory of differential equations; (5) On some points in the theory of continuous groups; by Lieut.-Col. A. CUNNINGHAM: Announcement of the discovery made jointly by Rev. J. Cullen, Mr. A. E. Western, and the author, of seven new factors of Fermat's numbers $(2^{2^n}+1)$; by Mrs. YOUNG: The surface representing all right-angled spherical triangles; by Mr. W. H. BUSSEY: Generational relations defining an abstract simple group of order 32,736; by Mr. W. H. YOUNG: (1) Skew surfaces contained in a linear congruence; (2) Closed sets of points and Cantor's numbers.

THE National academy of sciences held its annual meeting at Washington, D. C., April 21–23. No mathematical papers were presented at this meeting. Officers for the ensuing year are

as follows: president, ALEXANDER AGASSIZ; vice-president, IRA REMSEN; foreign secretary, SIMON NEWCOMB; home secretary, ARNOLD HAGUE; treasurer, S. F. EMMONS. Professor NEWCOMB was also appointed delegate to represent the Academy at the meeting of the international association of academies held at London in June. Professor E. PICARD, of the University of Paris, was elected foreign associate of the Academy.

AT its second general meeting, held at the Armour Institute of Technology, Chicago, April 10–11, the Central association of physics teachers was reorganized as the Central association of science and mathematics teachers. The mathematical section held two sessions, devoted to the discussion of improved methods of teaching elementary mathematics. One of the sessions was held jointly with the Chicago Section of the AMERICAN MATHEMATICAL SOCIETY, as noted in the Secretary's report, page 533 of the present number of the BULLETIN.

THE German mathematical society is making active preparations for the third international congress of mathematicians to be held at Heidelberg, August 8–13, 1904, under the presidency of Professor H. WEBER. The congress will divide into six sections devoted respectively to arithmetic and algebra, analysis, geometry, applied mathematics, history of mathematics, and teaching of mathematics. Exhibitions of models and of mathematical literature have been added to the programme. The government of Baden has appropriated 3,000 marks toward the financial support of the congress. Cards entitling the holder to general participation in the various events of the congress will be issued on payment of 20 marks. Each holder is then entitled to obtain additional cards for members of his family for 10 marks each. A daily bulletin will be issued during the meeting, giving besides the day's programme a list of those in attendance and their lodgings.

THE universities below offer during the academic year 1903–1904 advanced courses in mathematics as follows:

COLUMBIA UNIVERSITY. — By Professor T. S. FISKE: Advanced calculus, three hours; Theory of functions of a complex variable, three hours. — By Professor F. N. COLE: Theory of groups, three hours; Theory of invariants, three hours. — By Professor R. S. WOODWARD: Advanced theoretical mechanics, two hours; Theory of the potential function, two hours; Mathe-

matical theory of elasticity, two hours. — By Professor D. E. SMITH: History of mathematics, two hours. — By Professor J. MACLAY: Application of the calculus to surfaces and curves in space, three hours. — By Professor C. J. KEYSER: Modern theories in geometry, three hours. — By Dr. G. H. LING: Infinite series and products, two hours. — By Dr. E. KASNER: Transformations and continuous groups, two hours.

JOHNS HOPKINS UNIVERSITY.— By Professor F. MORLEY: Higher geometry, three hours; Vector analysis, two hours, first half year; Kinematics, two hours, second half-year. — By Dr. A. COHEN: Partial differential equations, two hours; Differential geometry, two hours, first half-year; Theory of numbers, two hours, second half-year; Elementary theory of functions, two hours.

THE memorial of the late Professor Sir G. G. STOKES, planned by the Royal Society and Cambridge University, is to take the form of a medallion portrait to be placed in Westminster Abbey.

THE Adams prize for 1903 (see BULLETIN, volume 7, page 321) was not awarded.

PROFESSOR M. NOETHER, of the University of Erlangen, has been elected corresponding member of the Paris academy of sciences.

PROFESSOR G. MITTAG-LEFFLER has been elected honorary member of the Finnish scientific society of Helsingfors.

THE Redpath professorship of mathematics at McGill University has been filled by the appointment of Professor JAMES HARKNESS, of Bryn Mawr College. Dr. H. M. TORY, of McGill University, has been promoted to an associate professorship of mathematics.

PROFESSOR FLORIAN CAJORI, of the department of mathematics at Colorado College, has been elected dean of the school of engineering of that institution. He has also been appointed one of the American representatives of the international commission on the organization of a congress of the history of sciences, referred to on page 509 of the current volume of the BULLETIN.

AT Purdue University Professor A. M. KENYON has been promoted to a full professorship of mathematics. Mr. W. H. BATES has been appointed instructor in mathematics.

AT the University of Nebraska, Dr. C. C. ENGBERG has been promoted to an adjunct professorship of mathematics.

AT Cornell University Dr. J. I. HUTCHINSON and Dr. VIRGIL SNYDER have been promoted to assistant professorships of mathematics.

MR. C. H. ASHTON has been appointed to an assistant professorship of mathematics in the University of Kansas.

MR. L. D. AMES has been appointed to an instructorship in mathematics in the University of Missouri.

MR. CLIFFORD GRAY and Mr. E. A. HOOK have been appointed instructors in mathematics at Columbia University.

DR. CARL STROMQUIST has been appointed to an instructorship in mathematics in the Greene School of Science of Princeton University.

MR. A. H. WILSON, of Princeton University, has received leave of absence and will spend the coming year at the University of Greifswald.

THE death of Professor C. A. BJERKNES, of the University of Christiania, is announced.

PROFESSOR L. GEGENBAUER, of the University of Vienna, died on June 3, at the age of fifty-four.

THE following catalogues of second-hand mathematical works have recently appeared: Burgersdijk and Niermans, Leyden, a few entries; Adolf Geering, Bäumleingasse 10, Basel, catalogue no. 175, a few entries; Ferdinand Schöningh, Lortzingstr. 2, Osnabrück, catalogue no. 43, 176 entries; W. Junk, Berlin, N. W., 5, a catalogue of 42 pages, representing a library of mathematical, physical, and astronomical works to be sold *en bloc* for 10,000 marks; Oskar Schack, Königsstrasse 15, Leipsic, catalogue no. 98, 401 entries in mathematics and physics; Paul Geuthner, 10 rue de Buci, Paris, catalogue no. 5, 241 entries, writers before 1800, also 91 portraits; Oskar Gerschel, 16 Calwerstr., Stuttgart, catalogue no. 27, 480 entries.

NEW PUBLICATIONS.

I. HIGHER MATHEMATICS.

ADAM (C.). See DESCARTES.

BIANCHI (L.). Lezioni sulla teoria dei gruppi continui di trasformazioni. Pisa, 1903. 8vo. 708 pp. (Lith.). Fr. 16.00

BOUCHER (M.). Essai sur l'hyperespace, le temps, la matière et l'énergie. Paris, Alcan, 1903. 16mo. 208 pp. Fr. 2.50

BÜRKLEN (O. T.). Formelsammlung und Repetitorium der Mathematik, enthaltend die wichtigsten Formeln und Lehrsätze der Arithmetik, Algebra, algebraischen Analysis, ebenen Geometrie, Stereometrie, ebenen und sphärischen Trigonometrie, mathematischen Geographie, analytischen Geometrie der Ebene und des Raumes, der Differential- und Integralrechnung. 2te Auflage, 4ter Abdruck. Leipzig, Göschen, 1903. 12mo. 229 pp. Cloth. (Sammlung Göschen, No. 51.) M. 0.80

CREFCOEUR (A. J. M.). Cours d'analyse. Calcul différentiel et calcul intégral. Méthode simple. Anvers, 1902. 8vo. 336 pp. Fr. 5.50

DELAPORTE (L. J.). Essai philosophique sur les géométries non-euclidiennes. (Thèse.) Paris, Naud, 1903. 8vo. 119 pp.

DESCARTES. Oeuvres de Descartes. Publiées par Charles Adam et Paul Tannery, sous les auspices du ministère de l'instruction publique. Vol. 5: Correspondance, mai 1647–février 1650. 669 pp. Vol. 6: Discours de la méthode et Essais. 12 + 730 pp. Paris, Cerf, 1902–1903. 4to.

DU BOIS (H. E. J. G.). Het verband tuschen wiskundige, proefondervindelijke en toegepaste natuurkunde. Utrecht, 1902. 8vo. 44 pp. M. 1.20

FORSYTH (A. R.). Treatise on differential equations. 3d edition. London, Macmillan, 1903. 8vo. 528 pp. Cloth. 14s.

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HOLST (E.). Om höiere arithmetiske raekker samt nogle af de almindeligst forekommende konvergerende raekker, med inledende saetninger om den hele funktion. 4te oplag. Christiania, 1902. 8vo. 55 pp. Boards. M. 1.50

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